

3.3.1: Number of research papers published per teacher in the Journals as notified on UGC CARE list during the last five years

3.3.1.1. Number of research papers in the Journals notified on UGC CARE list year wise during the last five years

CALENDAR YEAR: 2023

Title of paper	Name of the author/s	Department of the teacher	Name of journal	Calendar Year of publication	ISSN number	Link to the recognition in UGC enlistment of the Journal /Digital Object Identifier (doi) number		
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REVIEW PAPER ON FIXED POINT THEORY IN COMPLEX VALUED METRIC SPACE

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ABSTRACT

In this paper, we review some papers related to fixed point theory in complex valued metric space using contractive conditions, rational inequality and common limit range property for two pairs of mappings deriving common fixed point results under a generalized altering distance functions, E.A and CLR property.

1. Introduction:

The idea of complex valued metric space was presented by **Azam et al. [1]**, demonstrating some fixed point results for mappings fulfilling a rational inequality in complex valued metric spaces which is the generalization of cone metric space . Since then, several papers have managed fixed point hypothesis in complex valued metric spaces (see [3– 11] and references in that). **Rao et al. [12]** started the concentrate of fixed point results on complex valued b -metric spaces, which was broader than the complex valued metric spaces [1].

Following this paper, a number of authors have demonstrated a few fixed point results for different mapping fulfilling a rational conditions with regards to complex valued b -metric spaces (see[13– 16]) and the related references there in. As of late, **Sintunavarat et al. [9, 10]**, **Sitthikul and Saejung [11]**, and **Singhet al.[8]** obtained basic fixed point results by supplanting the consistent of contractive condition to control functions in complex valued metric spaces. In a continuation of [8,11,15,17], some normal fixed point results for a couple of mappings fulfilling more broad contractive conditions including rational expressions having point-subordinate control functions as coefficients in complex valued b -metric spaces have been proved by many authors.

2. Preliminaries:

Banach fixed point theorem [1] in a complete metric space has been summed up in numerous spaces. In 2011, **Azam et al. [2]** presented the thought of complex-valued metric space and built up sufficient conditions for the presence of common fixed points of a pair of mappings fulfilling a contractive condition. The possibility of complex-valued metric spaces can be abused to define complex-valued normed spaces and complex-valued Hilbert spaces; moreover it offers various research exercises in numerical examination. The theorems demonstrated by **Azam et al. [2]** and **Bhatt et al. [18]** utilize the rational inequality in a complex-valued metric space as contractive condition. In this paper, we present the idea of property (E.A) in a complex-valued metric space, to demonstrate some normal fixed point

Results for a fourfold of self-mappings fulfilling a contractive condition of 'max' type. Our outcomes sum up different theorems of customary metric spaces.

An ordinary metric d is a real-valued function from a set $X \times X$ into \mathbb{R} , where X is a nonempty set. That is, $d: X \times X \rightarrow \mathbb{R}$. A complex number $z \in \mathbb{C}$ is an ordered pair of real numbers, whose first co-ordinate is called $\text{Re}(z)$ and second coordinate is called $\text{Im}(z)$. Thus a complex-valued metric d is a function from a set $X \times X$ into \mathbb{C} , where X is a nonempty set and \mathbb{C} is the set of complex number. That is, $d: X \times X \rightarrow \mathbb{C}$. Let $z_1, z_2 \in \mathbb{C}$, define a partial order - on \mathbb{C} as follows:

$z_1 \preceq z_2$ if and only if $\text{Re}(z_1) \leq \text{Re}(z_2)$, $\text{Im}(z_1) \leq \text{Im}(z_2)$.

It follows that $z_1 \preceq z_2$ if one of the following conditions is satisfied:

(i) $\text{Re}(z_1) = \text{Re}(z_2)$, $\text{Im}(z_1) < \text{Im}(z_2)$,

(ii) $\text{Re}(z_1) < \text{Re}(z_2)$, $\text{Im}(z_1) = \text{Im}(z_2)$,

(iii) $\text{Re}(z_1) < \text{Re}(z_2)$, $\text{Im}(z_1) < \text{Im}(z_2)$,

(iv) $\text{Re}(z_1) = \text{Re}(z_2)$, $\text{Im}(z_1) = \text{Im}(z_2)$.

In (i), (ii) and (iii), we have $|z_1| < |z_2|$. In (iv), we have $|z_1| = |z_2|$. So $|z_1| \leq |z_2|$. In particular, $z_1 \prec z_2$ if $z_1 \neq z_2$ and one of (i), (ii), (iii) is satisfy. In this case $|z_1| < |z_2|$. We will write $z_1 < z_2$ if only (iii) satisfy. Further,

$0 \preceq z_1 \prec z_2 \Rightarrow |z_1| < |z_2|$,

$z_1 \preceq z_2$ and $z_2 < z_3 \Rightarrow z_1 < z_3$.

Azam et al. [2] defined the complex-valued metric space (X, d) in the following way:

Lenition 1.1. Let X be a nonempty set. Suppose that the mapping $d: X \times X \rightarrow \mathbb{C}$ satisfies the following conditions:

(C1) $0 \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;

(C2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(C3) $d(x, y) \preceq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a complex-valued metric on X , and (X, d) is called a complex valued metric space.

(i). Common Fixed Point Theorems Using Property (E.A) in Complex-Valued Metric Spaces.

Fixed Point Theorem Using (E.A)-Property [19]

In this paper author proved some important fixed point theorems using (E.A) property and (CLR) property in complex valued metric space in which the author also used the notion of partial order.

Theorem[a] Let (X, d) be a complex-valued metric space and $A, B, S, T: X \rightarrow X$ be four self-mappings satisfying:

- (i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$,
- (ii) $d(Ax, By) \leq k \max(d(Sx, Ty), d(By, Sx), d(By, Ty))$, $\forall x, y \in X$, $0 < k < 1$,
- (iii) the pairs (A, S) and (B, T) are weakly compatible,
- (iv) One of the pair (A, S) or (B, T) satisfy property (E.A).

If the range of one of the mappings $S(X)$ or $T(X)$ is a complete subspace of X , then mappings A , B , S and T have a unique common fixed point in X .

Fixed Point Theorem Using (CLR)-Property

The notion of (CLR)-property was defined by Sintunavarat and Kumam [20] in a metric space for a pair of self-mappings, which have the common limit in the range of one of the mappings.

Definition: (The (CLR)-property [20]). Suppose that (X, d) is a metric space and $f, g : X \rightarrow X$. Two mappings f and g are said to satisfy the common limit in the range of g property if $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = g x$, for some $x \in X$.

In the complex-valued metric space, the definition will be same but the space X will be a complex valued metric space.

Theorem[b]. Let (X, d) be a complex-valued metric space and $A, B, S, T : X \rightarrow X$ be four self-mappings satisfying:

- (i) $A(X) \subseteq T(X)$,
- (ii) $d(Ax, By) \leq k \max(d(Sx, Ty), d(By, Sx), d(By, Ty))$, $\forall x, y \in X$, $0 < k < 1$,
- (iii) the pairs (A, S) and (B, T) are weakly compatible.

If the pair (A, S) satisfy (CLR_A) property, or the pair (B, T) satisfy (CLR_B) property, then mappings A, B, S and T have a unique common fixed point in X .

(ii).Some fixed point theorems in complex valued metric spaces [11]

In this paper author proved several fixed point theorems for mappings satisfying certain point- dependent contractive conditions by deducing the results of [7] and [9] , [21].

Theorem: let, (X, d) be a complete complex valued metric space and $S, T : X \rightarrow X$. if there exists a mapping $\Lambda, \Xi : X \rightarrow [0, 1)$ such that for all $x, y \in X$:

- (i) $\Lambda(Sx) \leq \Lambda(x)$ and $\Xi(Sx) \leq \Xi(x)$;
 - (ii) $\Lambda(Tx) \leq \Lambda(x)$ and $\Xi(Tx) \leq \Xi(x)$;
 - (iii) $(\Lambda + \Xi)(x) < 1$;
 - (iv) $d(Sx, Sy) \leq \Lambda(x)d(x, y) + \frac{\Xi(x)d(x, Sx)d(y, Ty)}{1 + (x, y)}$
- then S and T have unique fixed point .

Cor: Let (X, d) be a complete complex valued metric space and $S, T : X \rightarrow X$. If there exist mappings $\lambda, \mu, \gamma : X \rightarrow [0, 1)$ such that for all $x, y \in X$:

(a) $\lambda(TSx) \leq \lambda(x)$, $\mu(TSx) \leq \mu(x)$ and $\gamma(TSx) \leq \gamma(x)$;

(b) $\lambda(x) + \mu(x) + \gamma(x) < 1$;

(c) $d(Sx, Ty) \leq \lambda(x)d(x, y) + \mu(x) \frac{d(x, Sx)d(y, Ty)}{1 + d(x, y)} + \gamma(x) \frac{d(x, Sx)d(y, Ty)}{1 + d(x, y)}$

Then S and T have a unique common fixed point.

Cor: If S and T are self-mappings defined on a complete complex valued metricspace (X, d) satisfying the condition

$$d(Sx, Ty) \leq \lambda d(x, y) + \mu \frac{d(x, Sx)d(y, Ty)}{1 + d(x, y)} + \gamma \frac{d(y, Sx)d(x, Ty)}{1 + d(x, y)}$$

for all $x, y \in X$, where λ, μ, γ are nonnegative reals with $\lambda + \mu + \gamma < 1$, then S and T have a unique common fixed point.

Cor: let, (X, d) be a real valued metric space. Let $T: X \rightarrow X$ be such that

(i) $d(Tx, Ty) \leq \lambda d(x, y) + \frac{\mu d(y, Ty)[1 + d(x, Tx)]}{1 + d(x, y)}$ for all $x, y \in X$, $\lambda > 0$, $\mu > 0$, $\lambda + \mu < 1$, and

(ii) for some $x_0 \in X$, the sequence of iterates $\{T^n(x_0)\}$ has a subsequence $\{T^{n_k}(x_0)\}$ with $z = \lim_{k \rightarrow \infty} T^{n_k}x_0$

Then z is a unique fixed point of T .

(iii). Six Maps with a Common Fixed Point in Complex Valued Metric Spaces [22]

In this paper, author attained a common fixed point theorem for six maps in complex valued metric space which is basically the generalization of [18]

Theorem: let (X, d) be a complex valued metric space and F, G, I, J, K, L be self maps of X satisfying the following conditions:

- (i) $KL(X) \subseteq F(X)$ and $IJ(X) \subseteq G(X)$
- (ii) $d(IJx, KLy) \leq ad(Fx, Gy) + b(d(Fx, IJx) + d(Gy, KLy)) + c(d(Fx, KLy) + d(Gy, IJx))$ for all $x, y \in X$, where $a, b, c \geq 0$ and $a + 2b + 2c < 1$. assume that the pairs (KL, G) and (IJ, F) are weakly compatible. pairs (K, L) , (K, G) , (L, G) , (I, J) , (I, F) and (J, F) are commuting pairs of maps. Then K, L, I, J, G and F have unique common fixed point in X .

(iv). Some Common Fixed Point Results for Rational Type Contraction Mappings in Complex Valued Metric Spaces [23]

In this paper, author demonstrates some fixed point theorems for two pairs which fulfil a rational type condition in complex valued metric space.

Fixed Point Theorem using E.A property

Theorem1: Let, (X, d) be a complex valued metric space and $A, B, S, T : X \rightarrow X$ four self-mappings satisfying the following conditions:

- (i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$
- (ii) For all $x, y \in X$ and $0 < a < 1$.

$$d(Ax,By) \leq a \frac{d(Sx,Ax)d(Sx,By)+d(Ty,By)d(Ty,Ax)}{1+(Sx,By)+d(Ty,Ax)}$$

- (iii) The pairs (A,S) and (B,T) are weakly compatible ;
- (iv) One of the pairs (A,S) or (B,T) satisfies (E.A) – property .
If the range of one of the mappings S(X) or T(X) is a closed subspace of X, then the mappings A,B,S and T have a unique common fixed point in X .

Theorem2: let, (X,d) be a complex valued metric space and A,B,S,T : X→ X four mappings satisfying the following conditions:

- (i) A(X) ⊆ T(X) , B(X) ⊆ S(X);
- (ii) For all x,y ∈ X and 0 < a < 1 ,

$$d(Ax,By) \leq \left\{ \begin{array}{l} a \frac{d(Sx, Ax)d(Sx, By) + d(Ty, By)d(Ty, Ax)}{d(Sx, By) + d(Ty, Ax)} \\ 0, \text{ if } D \neq 0 \\ \text{if } D = 0 \end{array} \right\}$$

- (iii) where D = d(Sx,By)+d(Ty,Ax);
- (iv) The pairs (A,S) AND (B,T) are weakly compatible ;
- (v) One of the pairs (A,S) or (B,T) satisfies (E.A)-property.
If the range S(X) or T(X) is a closed subspace of X, then the mappings A,B,S and T have unique common fixed point in X.

Fixed point theorem using (CLR)-property

Theorem3: let , (X,d) be a complex valued metric space and A,B,S and T : X→X four self-mappings satisfying the following conditions :

- (i) A(X) ⊆ T(X) , B(X) ⊆ S(X)
- (ii) For all x,y ∈ X and 0 < a < 1.

$$d(Ax,By) \leq a \frac{d(Sx,Ax)d(Sx,By)+d(Ty,By)d(Ty,Ax)}{1+(Sx,By)+d(Ty,Ax)}$$

- (iii) The pairs (A,S) and (B,T) are weakly compatible ;
the pair (A,S) satisfies CLR_A or (B,T) satisfies CLR_B – property .

If the range of one of the mappings S(X) or T(X) is a closed subspace of X, then the mappings A,B,S and T have a unique common fixed point in X .

Theorem4 : let (X,d) be a complex valued metric space and $A,B,S,T : X \rightarrow X$ four mappings satisfying the following conditions:

- (i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$;
- (ii) $d(Ax,By) \leq a d(Sx,By) + d(Ty,Ax)$ For all $x,y \in X$ and $0 < a < 1$,

$$d(Ax,By) \leq \begin{cases} a \frac{d(Sx,Ax)d(Sx,By) + d(Ty,By)d(Ty,Ax)}{d(Sx,By) + d(Ty,Ax)} \\ 0, \text{ if } D \neq 0 \\ \text{if } D = 0 \end{cases}$$

where $D = d(Sx,By) + d(Ty,Ax)$;

- (iii) The pairs (A,S) AND (B,T) are weakly compatible ;
If the pair (A,S) satisfies CLR_A or (B,T) satisfies CLR_B -property.
If the range $S(X)$ or $T(X)$ is a closed subspace of X , then the mappings A,B,S and T have unique common fixed point in X .

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